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THE MOTION OF A BODY THROUGH A LARGE-SCALE INHOMOGENEITY IN A STRATIFIED ATMOSPHERE†

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The free motion of a blunted body of revolution of considerable length through a large-scale cloud of heated gas (a thermal) floating in a stratified atmosphere is investigated. An effective numerical method of determining the trajectory of the body is proposed, based on the simultaneous solution of flow and ballistics problems. The flow problem is solved by a numerical method that is very economical in computer time, based on an expansion in a small parameter [1, 2], which is the angle of attack, in conjunction with the global-iteration method [2, 3]. The effect of the floating large-scale cloud of heated gas on trajectory, spatial orientation and stability of the body is determined. It is shown that under certain flight conditions the presence of a floating cloud of reduced density along the path of the body may lead to flipping of the body.

When the motion of a body in the atmosphere is being investigated it is necessary to determine the trajectory, which depends on the values of the aerodynamic coefficients, which are found by solving the problem of the flow around the body. At the same time, the solution of the problem of the flow around the body depends on the body orientation in space and the parameters of the incident flow, determined by the trajectory. To solve this problem approximately one usually solves only the systems of equations to determine the motions of the body in space (the ballistics problem), while to determine the current values of the aerodynamic coefficients one makes use of either a data bank, which has already been calculated, or one carries out calculations using approximate formulae.

To solve this problem with a high degree of accuracy one has to integrate simultaneously both the equations of motion of the body and the equations describing the motion of the gas around the body. This approach requires considerable computer resources, since at each time step one has to solve a complex system of equations to determine the aerodynamic coefficients. The integration step must then be chosen to be fairly small (of the order of the characteristic time in which the angle of attack changes). The latter approach can be improved considerably in the case of the supersonic motion of an elongated body of revolution. Such free motion is usually accompanied by small angles of attack.

To solve the equations of the motion of the gas around the body it is suggested below that the small-parameter method [1, 2] should be used, where the small parameter is the angle of attack. Then, for fixed conditions of the incident flow, a single-parameter family of solutions is obtained corresponding to different angles of attack. In other words, the dependence on the angle of attack using this approach is found analytically. In view of the fact that the characteristic time during which the angle of attack changes is usually considerably less than the characteristic time in which the parameters of the incident flow change, this approach enables the flow problem to be solved much more rarely. The method proposed enables one to determine the aerodynamic coefficients, the trajectory of motion and the body orientation with a high degree of accuracy, and also with the minimum computer costs for such an accuracy.

1. FORMULATION OF THE PROBLEM

We will assume that, at the initial instant of time in the stratified atmosphere of the Earth, a cloud of heated gas is formed having the following parameters

$$T(z,r) = T_a(z) + (T_{\max} - T_a(z))\exp\{-(R_T^{-1}l)^2\} \tag{1.1}$$

where $R_t = 4$ km, $T_{\max} = 10^4$ K, $T_a(z)$ is the temperature of the unperturbed atmosphere at an altitude z, and l is the distance from the centre of the spherical volume. The position of the temperature maximum corresponds to an altitude of $H = 20$ km.

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Due to the action of Archimedes forces the cloud floats up, forming a vortex ring. This is accompanied by intense turbulent mixing of the cold and hot layers of air. Fifteen seconds after the cloud begins to float up at an altitude of about 22 km the body enters it with a velocity of 2000 m/s. The body has the form of a cone with a blunt spherical head of radius $R_0 = 0.1$ m, a semi-aperture angle of 15°, a length of 2 m and a mass of 1 t, the centre of mass of the solid being situated a distance L from the vertex. The instant at which the solid is 8300 m from the axis of symmetry of the thermal is taken as the time origin. The body moves in a plane passing through the axis of symmetry of the thermal and the point at which the body is situated at the initial instant.

At the instant the body enters, the gas is twisted into a toroidal vortex ring, which floats up into the atmosphere. In view of this the flow around the body has a varying space–time structure. We will assume that the body has no effect on the gas motion in the thermal.

2. A NUMERICAL METHOD OF SOLVING THE PROBLEM

A numerical method of calculating the convective-diffusion of air in the region of a thermal is described in [4], where the motion of a blunt body along a specified rectilinear trajectory was also investigated.

The motion of the gas around the body was described by the system of complete equations of a viscous shock layer. It was assumed that the gas flow in the shock layer between the surfaces of the solid and the shock wave was quasi-steady at each point of the trajectory, since the characteristic time of motion of a fluid particle in a shock layer is much less than the characteristic time of variation of the parameters of the problem.

The small-parameter method was used to solve the system of equations of motion [1, 2]. The method consists of expanding the required spatial solution in an asymptotic series in the angle of attack. The expansion coefficients of powers of α (where α is the angle of attack) are then decomposed into a formal Fourier series in the meridional angle. Only the first term of the expansion in α is retained. The required solutions are then represented in the form $\Phi = \Phi_0 + \alpha\Phi_1\cos\psi$, where ψ is the meridional angle.

By using the small-parameter method one obtains the values of the aerodynamic coefficients in the following form: the drag coefficient $C_x = C_{x_0}$, the buoyancy coefficient $C_y = \alpha C_{y_1}$ and the pitching moment $M_z = \alpha M_{z_1}$. Here C_{x_0}, C_{y_1}, M_{z_1} are independent of α.

The coefficients of the Fourier expansion Φ_0 and Φ_1 were determined by the global-iteration method [2, 3].

The trajectory of motion of the body and its orientation in space is defined by the following system of equations [5]

$$\frac{d}{dt}\boldsymbol{\xi} = \mathbf{F}(\boldsymbol{\xi}, \mathbf{Q}, \mathbf{A}) \tag{2.1}$$

where

$$\boldsymbol{\xi} = \begin{Vmatrix} r \\ \varphi \\ \upsilon \\ \theta \\ \beta \\ \gamma \end{Vmatrix}, \quad \mathbf{Q} = \begin{Vmatrix} W_x \\ W_y \\ \rho \\ T \end{Vmatrix}, \quad \mathbf{A} = \begin{Vmatrix} C_x \\ C_{y_1} \\ M_{z_1} \end{Vmatrix}, \quad \gamma = \dot{\beta}$$

$$\mathbf{F} = \begin{Vmatrix} \upsilon\sin\theta \\ \upsilon r^{-1}\cos\theta \\ -g\sin\theta - \dfrac{S}{2m}\rho V^2 C_x \\ \left(\dfrac{\upsilon}{r} - \dfrac{g}{\upsilon}\right)\cos\theta + \dfrac{S}{2m}\rho V C_y \\ \gamma \\ -\dfrac{Sl^2}{2J_z}\rho V M_z^{\omega}\gamma - \dfrac{Sl}{2J_z}\rho V^2 M_z \end{Vmatrix}$$

Here r and φ are the polar coordinates of the centre of mass of the body, the pole of the system of coordinates coincides with the centre of the Earth, the polar axis is the Y-axis of a Cartesian system of coordinates, the X, Y plane of which coincides with the plane of motion of the centre of mass of the body, where the Y-axis passes through the centre of the Earth and the position of the centre of mass of the body at the initial instant of time, the angle φ is measured in a clockwise direction, υ is the modulus of the velocity vector $\mathbf{v}$ of the centre of mass of the body, θ is the angle between the vectors $\mathbf{v}$ and $\mathbf{r}_\varphi$, measured from the latter in an anticlockwise direction, r is the radius-vector of the centre of mass of the body, $r = |\mathbf{r}|$, the subscript z denotes that the corresponding quantity is calculated with respect to the axis passing through the centre of mass of the body, perpendicular to the plane of the trajectory of motion, m, S, l and J_z are the mass, the maximum cross-section area, the length and moment of inertia of the body, V is the modulus of the velocity of the incident flow with respect to the body, ρ is the air density in the incident flow, C_x, C_y, M_z are the aerodynamic coefficients, where $C_y = \alpha C_{y_1}$, $M_z = \alpha M_{z_1}$; M_z^ω is the aero-dynamic damping moment and β is the angle, measured in an anticlockwise direction, between the axis of symmetry of the body and the direction of the X-axis.

The vector ξ completely describes the body motion in space, and the vector $\mathbf{Q}$ has its own components required to calculate the parameters of the thermal (W_x and W_y are the components of the flow velocity in the thermal along the X and Y coordinate axes, respectively, ρ is the density and T is the air temperature at the point of space and the instant of time considered). Here $\mathbf{A} = \mathbf{A}(\xi, \mathbf{Q})$, $\mathbf{Q} = \mathbf{Q}(\xi, t)$; the vector $\mathbf{Q}$ is time-dependent due to the motion of the thermal.

The angle of attack is calculated from the formula

$$\alpha = \beta + \arcsin\{V_y V^{-1}\}$$

where V_y is the projection of the velocity vector $\mathbf{V}$ of the incident flow onto the Y-axis.

For simplicity, the values of the quantities φ, θ, β, $\dot\beta$ are taken to be zero at the instant $t = 0$.

In this formulation of the problem, the quantity M_z^ω can be neglected in view of its smallness.

The whole system of governing equations can be solved as follows.

At the initial instant, the system of equations of a viscous shock layer is solved for the specified parameters of the incident flow. Then, during a time Δt_g, the system of ballistics equations is integrated from the definition of the position of the centre of mass and the body orientation in space with fixed values of C_{x_0}, C_{y_1} and M_{z_1}, which depend only on the parameters of the incident flow, but with variable values of the angle of attack α. The step Δt_g is determined by the characteristic time of variation of the parameters of the incident flow. Here the system of ballistics equations is integrated with step $\Delta t_b \ll \Delta t_g$. This is due to the fact that the characteristic time of variation of the angle of attack is much less than the characteristic time of variation of the parameters of the incident flow (the body oscillates about the position corresponding to zero angle of attack). After a time Δt_g the sequence of the solution of the whole system is repeated with, possibly, other values of Δt_b and Δt_g.

The system of equations (2.1) is integrated as follows.

Consider system (2.1) in the time interval $[t^n, t^{n+1}]$, where $t^{n+1} = t^n + (\Delta t_g)^n$. We will assume that at the instant t^n, we know the values of all the quantities in (3.2) and the derivatives with respect to time of the vectors ξ and $\mathbf{Q}$.

We replace system (2.1) in the interval $[t^n, t^{n+1}]$ by the system

$$\begin{aligned} &\frac{d}{dt}\xi = \mathbf{F}(\xi, \mathbf{G}^n(t), \quad \mathbf{U}^n(t)) \\ &\mathbf{U}^n(t) = \mathbf{A}^n + \frac{\mathbf{A}^{n+1} - \mathbf{A}^n}{(\Delta t_g)^n}(t - t^n), \quad \mathbf{A}^{n+1} = \mathbf{A}(\xi_0^{n+1}, \ \mathbf{Q}(\xi_0^{n+1}, t^{n+1})) \end{aligned} \tag{2.2}$$

where $\mathbf{G}^n(t)$ is the local spline, which approximates the function $\mathbf{Q}(t)$ in the interval $[t^n, t^{n+1}]$, constructed from the values of the function $\mathbf{G}(t)$ and its derivatives at the points t^n and t^{n+1}, and ξ_0^{n+1} is found from the solution of the Cauchy problem

$$\frac{d}{dt}\xi_0 = \mathbf{F}(\xi_0, \mathbf{Q}_0^n(t), \mathbf{A}^n), \quad \xi_0(t^n) = \xi(t^n)$$

$$\mathbf{Q}_0^n(t) = \mathbf{Q}^n + \frac{d}{dt}\mathbf{G}^{n-1}(t^n)\cdot(t - t^n)\frac{d^2}{dt^2}\mathbf{G}^{n-1}(t^n)\cdot(t - t^n)^2$$

at the point t^{n+1}.

The solution of system (2.2) with accuracy $O(\Delta t_g^2)$ gives the solution of the initial system (2.1). System (2.2) is solved by the method of successive approximations and a solution is obtained with an error of less than 1% after 2–4 iterations. At each iteration of the method of successive approximations, the system of equations (2.2) was integrated by the Runge–Kutta method of the third order accuracy in Δt_b. The characteristic ratio $\Delta t_b/\Delta t_g$ here was 10^{-2}–10^{-3}. The time step Δt_g took values of 0.2–1 s.

The characteristic time taken to calculate the interval of physical time Δt_g amounted to several dozen minutes on an IBM 386/387 computer.

3. RESULTS

The gas in the region of the thermal, due to the action of Archimedes forces, is converted into a vortex ring. Here the thermal, as a whole, floats up, and the velocity field in it resembles a torus.

In the unperturbed atmosphere the centre of pressure turns out to be behind the centre of mass, and the body motion is stable. When the body moves towards the centre of the thermal the density of the incident flow falls rapidly, while the temperature increases, which leads to a reduction in the Mach number M_∞ and the Reynolds number Re_∞. Below we give data on the gas temperature T_∞, the gas flow velocity in the thermal W and the angle ϑ of inclination of the vector $\mathbf{W}$ to the axis of symmetry of the thermal at several points of the trajectory (the angle is measured from the axis of the axis of symmetry of the thermal, pointing upwards, in an anticlockwise direction); from the parameters of the incident flow the values of M_∞ and Re_∞ are determined, which are also given in Table 1.

A reduction in the number M_∞ and Re_∞ should cause the centre of pressure to shift towards the vertex of the cone. At the same time, a reduction in Re_∞ for fixed M_∞ shifts the centre of pressure in the opposite direction, as can be seen from Fig. 1. Nevertheless, the overall effect of a change in M_∞ and Re_∞ along the trajectory is to displace the centre of pressure towards the vertex of the cone, which reduces its stability.

In Fig. 1 we show a graph of the position of the centre of pressure L_p, measured from the vertex of the cone, as a function of the flight time. Here and henceforth the continuous curve corresponds to transit through an unperturbed atmosphere, while the dashed curve corresponds to transit through a thermal. The increase in L_p after leaving the thermal with respect to the unperturbed case can be explained by the relative increase in the velocity of the body, since the body is passing through a less dense medium.

During motion the body executes oscillations about the position corresponding to zero angle of attack. On passing through an unperturbed atmosphere the amplitude of the oscillations of the angle of attack is small (Fig. 2). We considered two cases of the position of the centre of mass: $L = 50$ cm (Fig. 2) and $L = 140$ cm (Fig. 3). When $L = 50$ cm the body has a considerable reserve of stability, and the oscillations of the angle of attack occur with relatively small amplitude. When $L = 140$ cm the angle of attack varies with a period and amplitude which is several times greater than for $L = 50$ cm. In the neighbourhood of the centre of the thermal, where the velocity of the gas motion is a maximum, the angle of attack is a maximum. In this region, as can be seen from Fig. 1, the centre of pressure turns out to be ahead of the centre of mass and the body position becomes unstable. This may cause the body to flip over.

In Figs 4 and 5 we also show graphs of the angle of inclination τ of the body axis to the horizontal (the pitching angle) for $L = 50$ cm and $L = 140$ cm, respectively. The maximum of τ in Fig. 4 is due to the fact that the body in this case, being extremely stable, turns around rapidly in the incident flow, which in the neighbourhood of the centre of the thermal has the maximum angle of inclination to the horizontal.

In Fig. 6 we show graphs of the trajectory of the solid for $L = 140$ cm. When $L = 50$ cm the presence of a thermal along the path of the body has practically no effect on the trajectory, and no change can be seen on the graph; in the second case, when $L = 140$ cm, the change in the trajectory is much more significant.

Table 1

t, s	2,0	3,2	3,8	4,2	4,7	5,3	6,6
T, K	220,2	420,5	623,8	663,0	614,4	440,2	219.0
w, m/s	15,4	60,1	184,1	235,6	199,1	79,2	17,6
ϑ, m/s	108,1°	26,7°	4,4°	0,0°	–3,2°	–15,3°	–145,6°
M_∞	6,44	4,53	3,81	3,74	3,88	4,61	6,18
$Re_\infty \times 10^5$	8,33	2,41	1,41	1,34	1,56	2,82	8,50

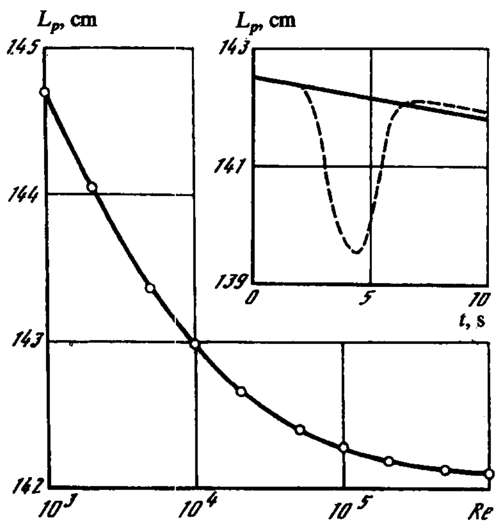

Fig. 1.

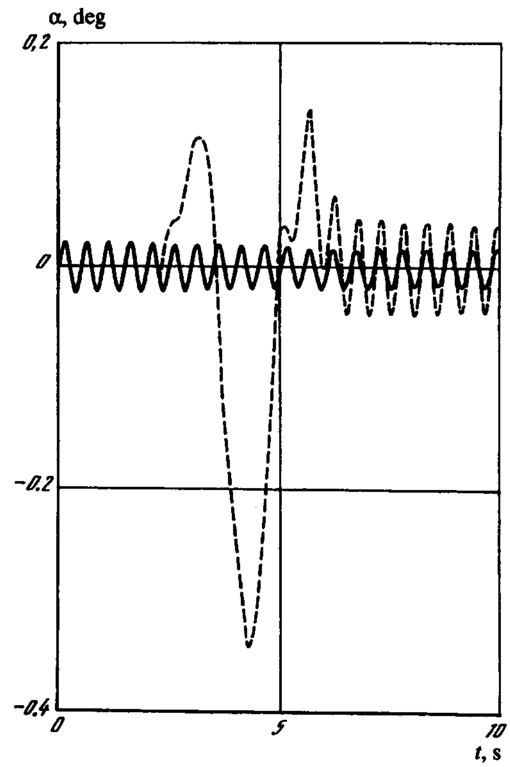

Fig. 2.

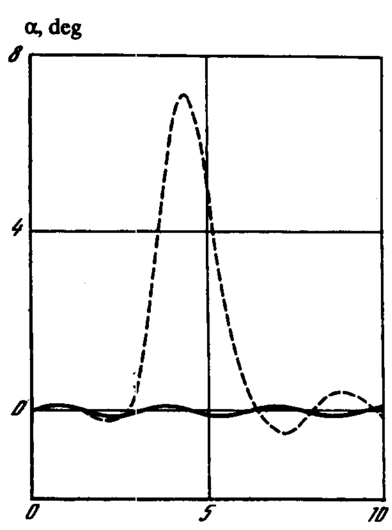

Fig. 3.

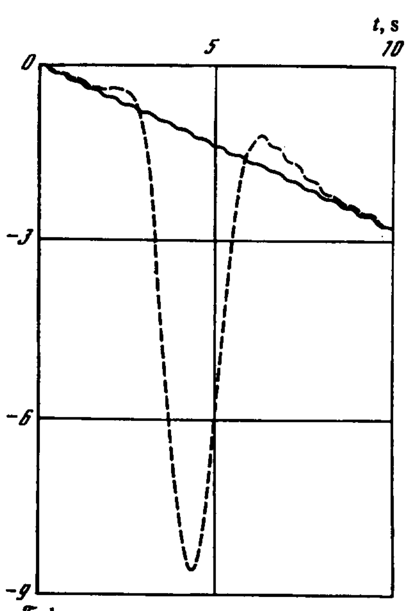

Fig. 4.

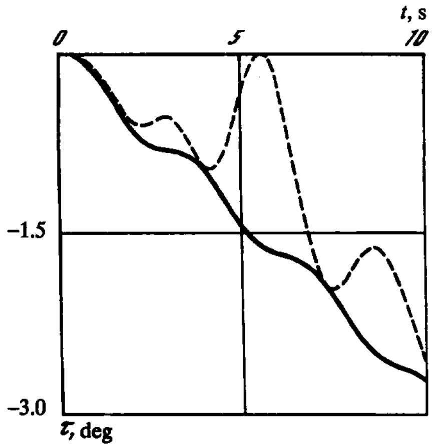

Fig. 5.

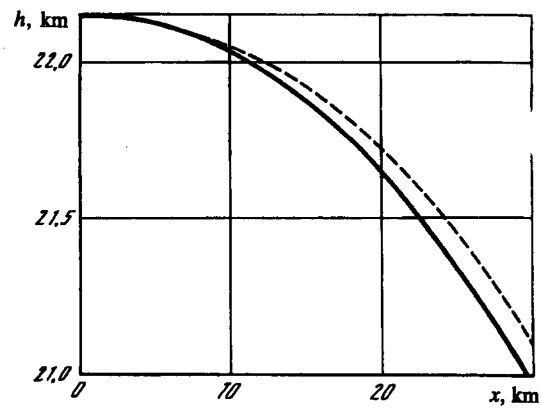

Fig. 6.

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